where $C = 3\left(\frac{s_*}{6} + A\right)A$, $A = \frac{1}{s_*} - \left(1 + \frac{1}{s_*^2}\right)\sin s_*$.

The neutral stability curves are shown in Fig. 4, where curves 1-5 correspond to S = 0, $0.7S_*$, S_*, $1.3S_*$, $2S_*$. We note that $Ra_* \approx 1301$ is greater than the minimum Rayleigh number at S = 0, which is approximately equal to 1100. Therefore at $S = S_*$ the point k = 0 is a local minimum and disturbances with k = 0 become most unstable for large S: $S \gtrsim 1.4S_*$.

From the cases analyzed here, we conclude that when the spirality increases from 0 to a certain value S_* the minimum of the neutral stability curves $Ra_0(k, S)$ shifts in the direction of smaller wave numbers k, and hence the horizontal dimensions of the convection cells increase. When $S \geqslant S_*$ the minimum is reached at k = 0 and the horizontal dimensions of the cells are limited by the external conditions (for example, inhomogeneities in the horizontal direction).

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EFFECT OF INTERPHASE MASS TRANSFER ON THE TURBULENCE ENERGY OF A FLOW OF A GAS SUSPENSION

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Semiempirical turbulence models based on equations describing the transfer of fluctuation energy are widely used to calculate flows of gas suspensions (see [1-3] and the accompanying bibliographies). Here, we attempt to use these models to describe the flows of gas suspensions with phase transformations (such as in [3], where allowance was made for the heterogeneous combustion of dispersed particles). We will analyze the direct effect of interphase mass transfer on the turbulence energy of the dispersion medium.

1. Equation of Turbulence Energy Transfer. In the presence of phase transformations, the equations of conservation of mass and momentum for the carrier phase (dispersion medium) are written as follows [4]

$$\partial\rho/\partial t + \nabla\cdot(\rho\mathbf{V}) = J; \tag{1.1}$$

$$\rho d\mathbf{V}/dt = -\nabla p + \nabla\tau - \mathbf{F} + J(V_p - V), \tag{1.2}$$

where ρ and $\mathbf{V}$ are the distributed density and velocity of the dispersion medium, the subscript p denotes the disperse phase, J is the intensity of the interphase mass transfer, p is pressure, τ is the shear stress, $\mathbf{F}$ is the interfacial force.

Using (1.1) and (1.2) and a well-known procedure (see [3], for example), we can obtain an equation for the fluctuation energy of the dispersion medium k. If we ignore fluctuations of the density of the gas ρ', this equation has the form

$$\rho\mathbf{V}\nabla k = \nabla[\mu\nabla k - \rho\langle\mathbf{V}'(\tfrac{1}{2}\mathbf{V}'^2 + p'/\rho)\rangle] - \rho\langle\mathbf{V}'\mathbf{V}'\rangle\nabla\mathbf{V} + \tfrac{1}{3}\mu\langle\mathbf{V}'\nabla(\mathrm{div}\ \mathbf{V}')\rangle + \langle p'\nabla\mathbf{V}'\rangle - \rho(\varepsilon + \varepsilon_p + \varepsilon_J) \tag{1.3}$$

Here and below, the primes denote the fluctuation component, while the remaining terms are averaged over time; $\rho\varepsilon = \mu\sum_{ij}\langle(\partial V_i'/\partial x_j)^2\rangle$ is the rate of viscous dissipation of the turbulence energy, μ is the coefficient of dynamic viscosity of the gas, $\rho\varepsilon_p = \sum_i\langle F_i'V_i'\rangle$ is a dissipative term due to the dynamic interaction of the phases and fluctuation motion.

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If we limit ourselves to the Stokes force among the forces associated with the phase interaction and we ignore ternary correlations, we can write the terms due directly to the effect of the particles in (1.3)

$$\rho\varepsilon_p = \beta\sum_i[(V_i - V_{pi})\langle\rho_p' V_i'\rangle + \rho_p\langle(V_i' - V_{pi}')\,V_i'\rangle]; \tag{1.4}$$

$$\rho\varepsilon_J = \sum_i[(V_i - V_{pi})\langle J' V_i'\rangle + J\langle(V_i' - V_{pi}')\,V_i'\rangle], \tag{1.5}$$

where $\beta = 18\mu/(\rho_p^0\delta^2)$ is the inverse of the dynamic relaxation time of a particle; and ρ_p^0 and δ are the true values of particle density and diameter.

It should be noted that Eq. (1.3) differs from that used in [2] in the presence of the term ε_J. This term is due directly to the effect of the phase transitions on turbulence energy and is found from Eq. (1.5). It can be seen from (1.4)-(1.5) that ε_p depends on the averaged and fluctuation values of phase velocity and distributed density for the disperse phase, while ε_J depends on the fluctuation and averaged velocities and the rate of the phase transitions. In the case when the rate of interphase mass transfer can be represented in the form $J = j(\rho_p + \rho_p')$, i.e., when the fluctuations of mass-transfer rate are due only to fluctuations of the concentration of the disperse phase, Eqs. (1.4) and (1.5) differ only in their coefficients. We represent their sum $\varepsilon_\Sigma = \varepsilon_p + \varepsilon_J$ as

$$\varepsilon_\Sigma = (1 + \gamma)\varepsilon_p, \quad \gamma = j/\beta. \tag{1.6}$$

It follows from Eq. (1.6) that the effect of mass transfer on turbulence energy can be compared with its dissipation on particles by means of the ratio γ. It should be noted that we are presently examining only the direct effect of interphase mass transfer on turbulence energy. The value of k may also be significantly affected by changes in the fields of averaged velocity and density of the gas.

If $\gamma > 0$, then ε_J helps reduce the fluctuation energy of the dispersion medium and vice versa. Let us examine the reasons for this effect. It is known (see [3], for example) that the level of fluctuation energy of the inertial dispersed impurity in the gas suspension is lower than the same energy for the dispersion medium; $J > 0$ means that the mass of the dispersed impurity is transferred to the dispersion medium with a lower level of turbulence energy, so that k decreases. If $J < 0$ in Eq. (1.1), the mass of the dispersion medium decreases, while that of the disperse phase increases. In this case, the turbulence energy of the dispersed impurity becomes greater than it should be for the given inertia. Thus, the turbulence energy of the dispersion medium is dissipated less in the dynamic interaction with the particles. The level of k is raised.

2. Gas-Dispersed Turbulent Flows with Interphase Mass Transfer. We will examine the isobaric turbulent uniform flow of a gas with monodisperse evaporating drops. The temperature of the drops reaches the boiling point at the given pressure $T_p = T_b$; the averaged velocities of the phases are the same. In this case, only the second terms are kept in the right sides of (1.4)-(1.5). The rate of interphase mass transfer [5]

$$J = [12\lambda/(\rho_p^0\delta^2 c_c)]\rho_p \ln(1 + \Delta T^*), \tag{2.1}$$

where $\Delta T^* = c_c(T - T_b)/\ell$ is the dimensionless temperature difference between the phases, c_c is the heat capacity of the vapor, ℓ is the heat of vaporization, g is the thermal conductivity.

At $\Delta T^* \ll 1$, we can write $\ln(1 + \Delta T^*) \approx \Delta T^*$ in a first approximation. Then, as can be seen from (2.1), the averaged value of the rate of interphase mass transfer should depend on the correlation $\langle\rho_p' T'\rangle$. However, it follows from physical considerations that the contribution of this correlation can be ignored compared to the contribution of the corresponding averaged term [3]. Thus, it follows from (2.1) that

$$\gamma = (2/3\mathrm{Pr})\ln(1 + \Delta T^*). \tag{2.2}$$

To evaluate the effect of mass transfer, we will follow [1] and approximate the Eulerian time correlation by the step function

$$R_E(t) = \begin{cases} 1, & t \leqslant t_E, \\ 0, & t > t_E \end{cases} \tag{2.3}$$

(t_E is the Eulerian integral turbulence scale). It was shown in [3] that when the averaged velocities of the phases are equal, it follows from (1.4) and (2.3) that

$$\varepsilon_p = 2k(\rho_p/\rho)\beta\exp(-\beta t_E). \tag{2.4}$$

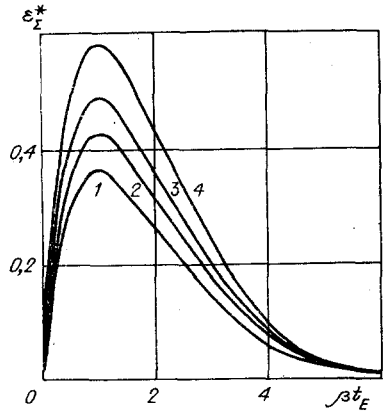

Fig. 1

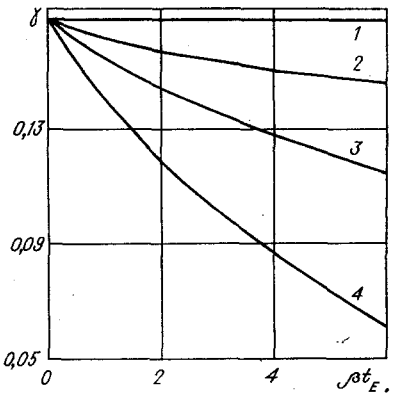

Fig. 2

Inserting (2.2) and (2.4) into (1.6), we obtain the following for the flow being examined

$$\varepsilon_\Sigma = 2k\frac{\rho_p}{\rho}\left(1 + \frac{2}{3\mathrm{Pr}}\ln(1 + \Delta T^*)\right)\beta\exp(-\beta t_E).$$

Figure 1 shows the change in the dimensionless quantity $\varepsilon_\Sigma^* = \varepsilon_\Sigma t_\Sigma/(2k)$ with Pr = 2/3 and $\rho_p/\rho = 1$ in relation to the relative inertia of a drop: 1) $\Delta T^* = 0$; 2) 0.2; 3) 0.4; 4) 0.8. It follows from (2.2) that γ is independent of the inertia of the particles but that ε_Σ is not. At $\beta t_E \to 0$, the high-inertia particles are not influenced by the fluctuation velocities of the gas. Thus, $\varepsilon_\Sigma \to 0$. At $\beta t_E \to \infty$, the fluctuation energy of the disperse phase is the same as that of the dispersion medium, and again $\varepsilon_\Sigma \to 0$. In these limiting cases, the effect of the disperse phase (including mass transfer) on the turbulence energy of the dispersion medium can be ignored for any ΔT^*. At intermediate values of βt_E, large values of ΔT^* lead to an increase in ε_Σ due to an increase in the rate J of the transfer of the low-turbulence mass of the disperse phase to the dispersion medium. For example, $\Delta T^* = 0.8$ for water drops at atmospheric pressure if the air temperature T = 1216 K.

We will examine turbulent gas flows with heterogeneous combustion and oxidation of the disperse particles. Let combustion take place in the diffusion regime with the formation of a gaseous oxide. Then the rate of interphase mass transfer $J > 0$, while its magnitude is determined [3, 6] as

$$J = A(b_0\rho_p + \langle b_0'\rho_p'\rangle),\ A = 6f\rho^0 D_0\mathrm{Nu}/(\rho_p^0\delta^2), \tag{2.5}$$

where f is the stoichiometric coefficient; D_0 and b_0 are the diffusion coefficient and mass fraction of oxygen.

It was shown in [3] that approximation (2.3) can be used to obtain an expression for the correlation

$$\langle b_0'\rho_p'\rangle = \rho_p b_0\varkappa(\exp(-Ab_0t_E) - 1),\ \varkappa = \langle b_0'^2\rangle/b_0^2 \tag{2.6}$$

Insertion of (2.6) into (2.5) yields $J = Ab_0\rho_p(1 + \varkappa(\exp(-Ab_0t_E) - 1))$. Then for the ratio γ we have

$$\gamma = B(1 + \varkappa(\exp(-B\beta t_E) - 1)),\ B = f\mathrm{Nu}b_0/3\mathrm{Sc}.$$

In contrast to the previous example, in this case γ depends on the relative inertia of the particles.

In the case of the reaction $C + O_2 = CO_2$ with heterogeneous combustion of dispersed particles of carbon, f = 3/8, and we take Nu = 2, Sc = 0.75. Figure 2 shows the dependence of γ on the relative inertia of the particles with $b_0 = 0.5$; 1) $\varkappa = 0$; 2) 0.2; 3) 0.5; 4) 1. It can be seen that γ is independent of βt_E when $\chi = 0$, while in the case of high-inertia particles ($\beta t_E \to 0$) it is independent of the level of fluctuation of oxygen concentration χ; at $\beta t_E \to \infty$ (low-inertia particles), $\gamma \to B(1 - \varkappa)$.

Now let metallic particles be oxidized in a high-temperature gas flow. The oxidation reaction is characterized by the formation of only condensed products on the surface of the particles (by scale formation). This does not prevent diffusion of oxygen to the metal, so that the oxidation rate depends only on temperature. According to [7], magnesium has these properties at 400-600°C.

In this case, $J < 0$, i.e., the mass of the disperse phase increases. The rate of interphase mass transfer is determined in the form

$$J = -6\rho_p/(\delta\rho_p^0)a, \quad a = K_0 \exp(-E/RT_p), \tag{2.7}$$

where a is the change in mass due to oxidation of a particle per unit time per unit of its area, K_0 is a pre-exponential multiplier, E is the activation energy, and R is the universal gas constant.

From (2.7), $\gamma = a\delta/(3\mu)$. Let us evaluate the upper boundary of this ratio. At 600°C in dry air, a $0.21 \cdot 10^{-4}$ kg/(m^2·sec) [7]; for the particle sizes $\delta = 10^{-5}$-10^{-4} m characteristic of power plants, the ratio $\gamma \approx 10^{-6}$-10^{-5} is so low that ε_J can be ignored in the equation for turbulence energy (1.3).

Thus, the direct effect of interphase mass transfer on turbulence energy must be considered in the case of intensive phase transformations.

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ONSET OF THERMOCAPILLARY CONVECTION IN A TWO-LAYER SYSTEM WITH THE RELEASE OF HEAT AT THE INTERFACE

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The onset of thermocapillary convection in a two-layer system with heating from below or above was studied in [1-4]. It was established that instability of the equilibrium state can result in both monotonic and oscillatory disturbances. Under certain conditions, only oscillatory instability is possible [3]. The presence of heat sources or sinks at the interface between the media – which may be due to a chemical reaction, absorption of radiation, etc. – has a significant effect on the stability of the system. The problem of the stability of the equilibrium state with surface heat release was solved in [5] in regard to monotonic disturbances.

Here, we study the effect of surface heat release and heat absorption on the stability of the equilibrium of a two-layer system in the presence of both monotonic and oscillatory instability. We will examine the evolution of oscillatory neutral curves for several characteristic cases. It is established that the heat release has a stabilizing effect on both monotonic and oscillatory disturbances.

1. Let the space between two horizontal solid plates – on which constant and different temperatures are maintained (temperature difference equal to θ) – be filled by two layers of viscous immicible fluids. The x axis is directed horizontally, while the y axis is directed

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